Hybrid Dense-Sparse Retrieval for Multi-hop Question Answering: A Complementary Scoring Approach

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***Abstract*—Multi-hop question answering systems require effective retrieval mechanisms to identify relevant documents from large-scale knowledge bases. While dense retrieval methods leveraging pre-trained language models have demonstrated superior semantic matching capabilities, they exhibit limitations in exact term matching and handling rare entities—critical components for multi-hop reasoning chains. This paper proposes a hybrid retrieval framework that synergistically combines dense neural representations with sparse lexical matching using BM25 probabilistic ranking. We introduce a normalized weighted scoring mechanism with adaptive parameter tuning to balance semantic similarity and lexical precision. Experimental evaluation on the HotpotQA benchmark demonstrates that our approach achieves a 3.4% improvement in top-1 retrieval accuracy and 2.9% in top-10 accuracy over the baseline Multi-hop Dense Retrieval (MDR) system, with minimal computational overhead (8-12ms per query). Error analysis reveals that the hybrid approach particularly excels in retrieving documents containing rare entities (42% of improvements), temporal expressions (23%), and technical acronyms (18%). Our findings suggest that incorporating complementary retrieval paradigms should be considered a fundamental component in multi-hop question answering architectures.**

***Index Terms*—Multi-hop Question Answering, Dense Retrieval, Hybrid Retrieval, Information Retrieval, Neural Networks, BM25**

I. Introduction

Multi-hop question answering represents a challenging natural language understanding task that requires systems to retrieve and reason over multiple interconnected documents to synthesize coherent answers [2]. Unlike single-hop factoid question answering, multi-hop QA necessitates the identification of bridging entities that connect disparate information sources, thereby enabling complex reasoning chains across multiple evidence documents. The Multi-hop Dense Retrieval (MDR) framework [1] introduced an iterative retrieval paradigm utilizing dense vector representations learned through transformer-based encoders, achieving state-of-the-art performance on benchmark datasets. However, despite their semantic understanding capabilities, dense retrieval methods face inherent limitations in matching exact lexical patterns and rare entity mentions—characteristics that are frequently essential for establishing connections in multi-hop reasoning scenarios [3].

Traditional sparse retrieval methods, exemplified by BM25 [4], employ term frequency-inverse document frequency statistics to rank documents based on lexical overlap. While these approaches lack semantic generalization capabilities and struggle with vocabulary mismatch problems, they demonstrate remarkable precision in exact term matching and provide computational efficiency through inverted index structures [5]. Recent research in single-hop dense passage retrieval has indicated that hybrid approaches combining dense and sparse signals can capture complementary aspects of relevance, leading to improved retrieval effectiveness [6], [7].

Despite these advances, the application of hybrid retrieval methodologies to multi-hop question answering remains relatively unexplored. The unique characteristics of multi-hop queries—including entity-centric reasoning, temporal constraints, and compositional semantics—motivate investigation into whether complementary retrieval paradigms can enhance document ranking in this domain. This paper addresses this gap by proposing a simple yet effective hybrid scoring mechanism that integrates dense neural retrieval with sparse lexical matching for multi-hop question answering.

Our contributions are threefold: (1) We design and implement a normalized hybrid scoring function that combines dense embeddings from pre-trained MDR models with BM25 lexical scores through weighted linear interpolation. (2) We conduct comprehensive empirical evaluation on the HotpotQA benchmark dataset, demonstrating consistent improvements across multiple retrieval metrics with minimal computational overhead. (3) We perform detailed error analysis to characterize the types of queries and document characteristics where hybrid retrieval provides the most substantial gains, offering insights into the complementary nature of dense and sparse retrieval paradigms in multi-hop settings.

II. Related Work

A. Multi-hop Question Answering

Multi-hop question answering has emerged as a critical research direction in natural language processing. The HotpotQA

dataset [2] established a large-scale benchmark containing questions requiring reasoning over multiple Wikipedia articles, with explicit supervision for supporting facts. The Multi-hop Dense Retrieval (MDR) framework [1] proposed an iterative retrieval approach where RoBERTa-based encoders [8] learn dense representations optimized for multi-hop reasoning. The system retrieves documents sequentially, with each retrieved document providing contextual information for subsequent retrieval steps. Training employs a momentum-based contrastive loss that maximizes similarity between query encodings and gold supporting documents while pushing apart negative samples. Recent extensions have explored graph-based reasoning [9], reinforcement learning for retrieval policy optimization [10], and unified retrieval-reading architectures [11].

B. Dense and Sparse Retrieval

Dense retrieval methods leverage pre-trained language models to encode queries and documents into fixed-dimensional vector spaces, enabling efficient similarity search through approximate nearest neighbor algorithms [3]. DPR [3] demonstrated that independently encoded query and document representations could outperform BM25 on open-domain question answering. ANCE [12] introduced asynchronous negative sampling and approximate nearest neighbor index updates during training. ColBERT [13] proposed late interaction mechanisms that preserve fine-grained matching signals while maintaining computational efficiency.

Sparse retrieval methods, particularly BM25 [4], remain competitive baselines due to their exact matching capabilities and interpretability. BM25 ranks documents using probabilistic term weighting that considers term frequency saturation and document length normalization. Despite lacking semantic understanding, BM25 demonstrates robustness across diverse domains and query types [5]. Recent learned sparse retrieval approaches [14], [15] attempt to combine the interpretability of sparse representations with neural learning, generating expanded sparse vectors that capture semantic relationships while maintaining term-level matching.

C. Hybrid Retrieval Approaches

Hybrid retrieval systems combine multiple retrieval paradigms to leverage complementary strengths. Luan et al. [6] proposed interpolating dense and sparse scores for first-stage ranking, demonstrating improvements on MS MARCO passage ranking. The E5 embedding model [7] incorporated both dense and sparse training objectives, though primarily focusing on dense representations for inference. Recent work in conversational search [16] has explored dynamic score combination based on query characteristics. However, these approaches have primarily targeted single-hop retrieval scenarios, and their applicability to multi-hop reasoning chains remains underexplored.

III. Methodology

A. Problem Formulation

Given a multi-hop question q and a corpus $\mathcal{D} = \{d_1, d_2, ..., d_N\}$ containing N documents, the retrieval task aims to identify a ranked list of k documents $\mathcal{R}_k = \{d_{r_1}, d_{r_2}, ..., d_{r_k}\}$ that maximize the probability of containing supporting evidence for answering q. In the multi-hop setting, the gold supporting set typically consists of two or more documents $\mathcal{G} = \{d_{g_1}, d_{g_2}, ..., d_{g_m}\}$ where $m \geq 2$. The retrieval quality is measured by the overlap between $\mathcal{R}_k$ and $\mathcal{G}$.

B. Dense Retrieval Component

The dense retrieval component employs the pre-trained MDR encoder architecture [1], which consists of two RoBERTa-base encoders: a query encoder $f_q : \mathcal{Q} \to \mathbb{R}^d$ and a document encoder $f_d : \mathcal{D} \to \mathbb{R}^d$, where $d = 768$ represents the embedding dimension. For a given query-document pair (q, d_i), the dense relevance score is computed as the dot product of their embeddings:

$$s_{\text{dense}}(q, d_i) = f_q(q)^T \cdot f_d(d_i) \tag{1}$$

The encoders are trained using momentum-based contrastive learning with in-batch negatives, optimizing the loss function:

$$\mathcal{L}_{\text{dense}} = -\log \frac{\exp(s_{\text{dense}}(q, d^+)/\tau)}{\sum_{d_j \in \{d^+\} \cup \mathcal{N}} \exp(s_{\text{dense}}(q, d_j)/\tau)} \tag{2}$$

where d^+ denotes a positive (gold) document, $\mathcal{N}$ represents negative samples, and τ is a temperature hyperparameter.

C. Sparse Retrieval Component

The sparse retrieval component utilizes the BM25 probabilistic ranking function [4], which computes relevance scores based on term frequency statistics. For a query q containing terms $\{t_1, t_2, ..., t_m\}$ and document d_i, the BM25 score is:

$$s_{\text{sparse}}(q, d_i) = \sum_{t \in q} \text{IDF}(t) \cdot \frac{\text{TF}(t, d_i) \cdot (k_1 + 1)}{\text{TF}(t, d_i) + k_1 \cdot (1 - b + b \cdot \frac{|d_i|}{\text{avgdl}})} \tag{3}$$

where $\text{TF}(t, d_i)$ denotes term frequency of t in document d_i, $\text{IDF}(t) = \log \frac{N - n(t) + 0.5}{n(t) + 0.5}$ is the inverse document frequency with $n(t)$ documents containing term t, $|d_i|$ represents document length, avgdl is the average document length in the corpus, and $k_1 = 1.5$, $b = 0.75$ are standard BM25 parameters controlling term frequency saturation and length normalization respectively.

D. Hybrid Scoring Function

Raw scores from dense and sparse retrieval components exist in different numerical ranges and distributions. Direct combination without normalization would result in one component dominating the final ranking. We therefore apply min-max normalization to standardize scores to the $[0, 1]$ interval:

$$s'_i = \frac{s_i - \min_j(s_j)}{\max_j(s_j) - \min_j(s_j) + \epsilon} \tag{4}$$

where $\epsilon = 10^{-10}$ prevents division by zero. The normalized scores are then combined using weighted linear interpolation:

$$s_{\text{hybrid}}(q, d_i) = \alpha \cdot s'_{\text{dense}}(q, d_i) + (1 - \alpha) \cdot s'_{\text{sparse}}(q, d_i) \quad (5)$$

where $\alpha \in [0, 1]$ controls the relative contribution of dense versus sparse retrieval. The hybrid score s_{hybrid} is used to rank all documents in the corpus, and the top-k documents are selected as the retrieval result.

E. Implementation Details

We implement the hybrid retrieval system by modifying the MDR evaluation pipeline. The BM25 index is constructed using the rank_bm25 Python library, which implements an efficient inverted index structure. Index construction occurs once as a preprocessing step, requiring approximately 2.3 minutes for the HotpotQA corpus (5.23M documents). At query time, we compute both dense and sparse scores in parallel. Dense score computation utilizes GPU acceleration through PyTorch, while BM25 scoring operates on CPU using the pre-built inverted index. The additional latency introduced by BM25 is minimal (8-12ms per query) compared to dense encoding (45ms). We perform min-max normalization over the top-10,000 candidates from each retriever before score combination to reduce computational cost while maintaining ranking quality.

IV. Experimental Setup

A. Dataset

We evaluate on HotpotQA [2], a large-scale multi-hop question answering benchmark built on Wikipedia articles. The dataset contains 112,779 question-answer pairs, with each question requiring reasoning over two supporting documents selected from 5.23 million Wikipedia paragraphs. Questions are categorized as bridge (requiring entity linking between documents) or comparison (requiring comparison of properties across entities). The validation set comprises 7,405 questions with gold supporting document annotations. We use the full Wikipedia corpus as our retrieval pool, making this a realistic open-domain retrieval scenario.

B. Baseline System

Our baseline is the official MDR implementation [1] using RoBERTa-base as the encoder backbone. The model was trained on the HotpotQA training set for 30,000 steps with batch size 128, learning rate 2×10^{-5}, and momentum coefficient 0.999 for the document encoder. We use the publicly released checkpoint which achieved state-of-the-art results on the HotpotQA leaderboard at the time of publication. For fair comparison, our hybrid approach uses identical dense retrieval components, adding only the BM25 sparse scoring.

C. Evaluation Metrics

We report the following metrics: (1) **Top-k Accuracy**: Percentage of questions where at least one gold supporting document appears in the top-k retrieved documents. We report results for $k \in \{1, 5, 10, 50\}$. (2) **Mean Reciprocal Rank (MRR)**: Average of $\frac{1}{\text{rank}}$ where rank is the position of the first gold document in the ranked list. (3) **Recall@k**: Proportion of all gold supporting documents that appear within the top-k retrieved documents. This is particularly relevant for multi-hop QA where multiple supporting documents must be retrieved.

D. Hyperparameter Selection

The primary hyperparameter is α, controlling the balance between dense and sparse scores. We perform grid search over $\alpha \in \{0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ using a held-out development set of 1,000 questions sampled from the training data. We select $\alpha = 0.7$ based on maximizing top-10 accuracy on this development set. All reported results use this fixed α value. We maintain all other hyperparameters from the baseline MDR system, including batch size (100), number of retrieved documents (100), and encoding parameters.

V. Results and Analysis

A. Main Results

Table I presents retrieval performance on the HotpotQA validation set. Our hybrid approach demonstrates consistent improvements across all evaluation metrics. Top-1 accuracy improves from 58.3% to 61.7%, representing a 3.4 percentage point absolute gain and 5.8% relative improvement. This indicates that the hybrid scoring mechanism more accurately identifies the most relevant document as the top-ranked result. Similar improvements are observed at higher cutoffs: top-5 accuracy increases by 3.3 points (78.5% to 81.8%), and top-10 accuracy improves by 2.9 points (84.2% to 87.1%). The MRR metric increases from 0.665 to 0.694, confirming that relevant documents are ranked higher on average. Recall@50 shows a more modest improvement of 1.8 points, suggesting that both retrievers identify similar documents at higher cutoffs, but the hybrid approach excels at re-ranking to promote relevant documents to top positions.

TABLE I
Retrieval Performance on HotpotQA Validation Set

Method	Top-1	Top-5	Top-10	MRR
MDR (baseline)	58.3	78.5	84.2	0.665
Hybrid (α=0.7)	**61.7**	**81.8**	**87.1**	**0.694**
Absolute Δ	+3.4	+3.3	+2.9	+0.029
Relative Δ	+5.8%	+4.2%	+3.4%	+4.4%

B. Ablation Study on Weighting Parameter

Table II analyzes the effect of the weighting parameter α on retrieval performance. Pure sparse retrieval ($\alpha = 0$) achieves 54.2% top-1 accuracy, confirming that BM25 alone is competitive but inferior to dense retrieval. Pure dense retrieval ($\alpha = 1.0$) corresponds to the baseline MDR performance. The hybrid approach demonstrates superior performance across a range of α values, with optimal results at $\alpha = 0.7$. This suggests that dense retrieval should be weighted more heavily (70%) than sparse retrieval (30%), consistent with prior work indicating that semantic matching is primary while lexical

matching serves a complementary role. Performance degrades gracefully as α deviates from the optimal value, indicating robustness to hyperparameter selection. The results validate that both components contribute meaningfully to the final ranking quality.

TABLE II
IMPACT OF WEIGHTING PARAMETER α

α	Top-1	Top-10	MRR
0.0 (sparse only)	54.2	82.8	0.638
0.5	60.1	85.8	0.681
0.6	61.2	86.5	0.688
0.7	**61.7**	**87.1**	**0.694**
0.8	61.3	86.7	0.689
0.9	60.5	85.9	0.683
1.0 (dense only)	58.3	84.2	0.665

C. Error Analysis

We performed manual analysis of 200 questions randomly sampled from cases where hybrid retrieval succeeded (retrieved at least one gold document in top-10) but baseline MDR failed. Four primary patterns emerged:

Rare Entity Mentions (42%): Questions containing uncommon person names, specific locations, or technical terminology where exact lexical matching is critical. Example: "Which genus of plants has more species, Graptopetalum or Dudleya?" The baseline dense retriever failed to match the rare botanical terms, while BM25 captured the exact string matches.

Temporal Expressions (23%): Questions requiring specific years, dates, or temporal ordering. Example: "What year did the author of 'The Curious Incident of the Dog in the Night-Time' publish their first novel?" Dense embeddings struggled to distinguish between different years mentioned in documents, while BM25 precisely matched the year values.

Acronyms and Abbreviations (18%): Questions containing technical acronyms that may not have been well-represented in pre-training data. Example: "Which NASA mission launched in 1997 studied Saturn?" The acronym "NASA" and the specific year "1997" benefit from exact matching.

Multi-word Expressions (17%): Questions with fixed phrases, proper nouns, or idioms where n-gram matching improves relevance. Example: "What is the birth name of the actor who played in 'The Shawshank Redemption'?" The quoted movie title requires precise multi-word matching.

These patterns indicate that the primary advantage of hybrid retrieval stems from BM25's precision in matching exact lexical patterns, particularly for entities and constraints that establish connections between documents in multi-hop chains.

D. Computational Efficiency

We measured retrieval latency on a system with Intel Xeon E5-2698v4 CPU, 256GB RAM, and NVIDIA V100 GPU. Dense encoding with the RoBERTa-base model requires 45ms per query (batch size 1) on GPU. BM25 scoring over the full corpus using the pre-built inverted index adds 8-12ms on CPU. Min-max normalization and score combination contribute negligible overhead (¡1ms). Total end-to-end retrieval time increases from 45ms to 53-57ms per query, representing 18-27% overhead. For batch processing with batch size 100, amortized per-query latency is 4.8ms for dense encoding and 9.2ms for hybrid retrieval (92% overhead). The BM25 index requires 2.1GB disk space for the HotpotQA corpus. These overheads are acceptable for most production scenarios, especially considering the substantial accuracy improvements.

VI. DISCUSSION AND LIMITATIONS

A. Why Hybrid Retrieval Works

The effectiveness of hybrid retrieval in multi-hop QA stems from the complementary nature of dense and sparse retrieval paradigms. Dense retrieval excels at semantic matching, capturing paraphrases, synonyms, and conceptual relationships between queries and documents. This is particularly valuable when question phrasing differs from document wording. However, dense encoders may conflate similar entities or fail to distinguish between specific numerical values, dates, or rare terms that were underrepresented in pre-training corpora. Sparse retrieval provides precision through exact term matching, which is critical for entity-centric multi-hop reasoning where bridging entities must be matched precisely to connect documents. The weighted combination allows the system to leverage semantic understanding for general relevance while relying on lexical precision for specific constraints.

B. Comparison with Alternative Approaches

Our approach differs from recent learned sparse retrieval methods like SPLADE [14] in that we use traditional BM25 rather than learned sparse representations. This design choice offers several advantages: (1) No additional training is required, enabling immediate application to new domains. (2) BM25 indices are standard components in search systems, facilitating practical deployment. (3) Computational requirements remain minimal compared to training and maintaining learned sparse models. However, learned sparse approaches may capture more sophisticated term relationships and could potentially achieve superior performance with domain-specific training.

C. Limitations and Future Work

Several limitations warrant discussion. First, the optimal α value may vary across datasets, domains, and query types. While we observe robust performance across a range of α values, adaptive or query-specific weighting mechanisms could provide additional gains. Second, our approach operates on the first-hop retrieval stage of MDR but does not modify the iterative retrieval process. Extending hybrid scoring to subsequent retrieval hops, where query context includes previously retrieved documents, represents promising future work. Third, BM25 requires preprocessing the entire corpus to build inverted indices, which may be infeasible for extremely large or dynamically updated corpora. Finally, our evaluation

focuses solely on HotpotQA; validation on additional multi-hop QA datasets would strengthen generalization claims.

Future research directions include: (1) Developing query-adaptive weighting mechanisms that adjust α based on query characteristics such as entity density, question type, or confidence scores from individual retrievers. (2) Investigating learned interpolation functions that replace fixed linear combination with trainable fusion networks. (3) Extending the hybrid approach to iterative multi-hop retrieval stages. (4) Evaluating on diverse multi-hop QA benchmarks including 2WikiMultihopQA and MuSiQue to assess cross-dataset generalization.

VII. Conclusion

This paper presented a hybrid retrieval framework for multi-hop question answering that combines dense neural representations with sparse lexical matching. Through normalized weighted scoring, we effectively leverage the complementary strengths of semantic similarity and exact term matching. Comprehensive evaluation on the HotpotQA benchmark demonstrated consistent improvements across multiple metrics, with top-1 accuracy increasing by 3.4 percentage points and minimal computational overhead. Error analysis revealed that hybrid retrieval particularly benefits queries involving rare entities, temporal expressions, and technical terminology—characteristics frequently present in multi-hop reasoning chains. Our findings suggest that hybrid retrieval should be considered a fundamental component in multi-hop question answering systems. The simplicity and effectiveness of the proposed approach, combined with its minimal implementation complexity, make it readily applicable to existing dense retrieval architectures. Future work will explore adaptive weighting mechanisms and extension to iterative multi-hop retrieval stages.

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